***Instrumental Variables*… A Way Too Simple Presentation**

The DGM: $Y = \beta_0 + \beta_1 X + U$

The Goal: Estimate β_1 with n independent observations $\{x_i, y_i\}$ $i = 1, \ldots n$

The Problem I: *X* and *U* are correlated… and so the OLS estimates are subject to O*mitted Variable Bias*, and accordingly, biased.

The Problem II: More specifically: $\text{cov}(X,U) \neq 0$.

Since $Y = \beta_0 + \beta_1 X + U$, $\text{cov}(X,Y) = \beta_1 \text{cov}(X,X) + \text{cov}(X,U)$.

- If $\text{cov}(X,U) = 0$, then we have $\beta_1 = \frac{\text{cov}(X,Y)}{\text{cov}(X,X)} - \frac{\text{cov}(X,U)}{\text{cov}(X,X)} = \frac{\text{cov}(X,Y)}{\text{cov}(X,X)}$, since $\text{cov}(X,U) = 0$. And so we can estimate β_1 with the usual ratio of *Sample Covariances*: $\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}}$.
- But if $\text{cov}(X,U) \neq 0$, then $\beta_1 = \frac{\text{cov}(X,Y)}{\text{cov}(X,X)} - \frac{\text{cov}(X,U)}{v\text{ov}(X,X} \neq \frac{\text{cov}(X,Y)}{\text{cov}(X,X)}$ and so $\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}}$ will not work.

The Solution: Find another variable Z such that $\text{cov}(Z,X) \neq 0$ and $\text{cov}(Z,U) = 0$.

Any Z correlated with X and uncorrelated with U will work.

Then looking at the covariance of Z with Y we have

$\text{cov}(Z,Y) = \beta_1 \text{cov}(Z,X) + \text{cov}(Z,U)$, which implies that $\text{cov}(Z,Y) = \beta_1 \text{cov}(Z,X)$ since $\text{cov}(Z,U) = 0$.

And since $\beta_1 = \frac{\text{cov}(Z,Y)}{\text{cov}(Z,X)} - \frac{\text{cov}(Z,U)}{\text{cov}(Z,X)} = \frac{\text{cov}(Z,Y)}{\text{cov}(Z,X)}$, since $\text{cov}(Z,U) = 0$, we can estimate β_1 with the following ratio of *Sample Covariances*: $\hat{\beta}_1 = \frac{S_{zy}}{S_{zx}}$.

Z is called an ***Instrumental Variable*** (for X).

***The Implementation*: TSLS (Two Stage Least Squares)**

1) Regress the x's on the z's using OLS.
2) Capture the predicted values , the $\hat{x}'s$
3) Then regress the y's on the predicted x's from the first regression
4) Your slope coefficient will be $\hat{\beta}_1 = \dfrac{S_{zy}}{S_{zx}}$

Outline of Proof:

From the 1st stage regression: $\hat{x}_i = \hat{\alpha} + \dfrac{S_{zx}}{S_{zz}} z_i$.

From the second stage regression, the OLS slope estimate will be the ratio of

$$SampleCov(y_i\text{'}s, \frac{S_{zx}}{S_{zz}} z_i\text{'}s) = \frac{S_{zx}}{S_{zz}} S_{zy} \text{ and } SampleVar(\frac{S_{zx}}{S_{zz}} z_i\text{'}s) = \left(\frac{S_{zx}}{S_{zz}}\right)^2 S_{zz} = \frac{S_{zx}^z}{S_{zz}}.$$

But we have lots of things cancelling in this ratio: $\dfrac{\frac{S_{zx}}{S_{zz}} S_{zy}}{\frac{S_{zx}^z}{S_{zz}}} = \dfrac{S_{zx}}{S_{zz}} S_{zy} \dfrac{S_{zz}}{S_{zx}^z} = \dfrac{S_{zy}}{S_{zx}}$.

So the slope estimate in the 2nd regression will give us the desired ratio of the sample covariances: $\hat{\beta}_1 = \dfrac{S_{zy}}{S_{zx}}$.

Two Issues:

- You never know for sure whether Z really is uncorrelated with U... because U is unknown.
- If the X and Y correlations with Z are very weak, it may be quite difficult to estimate them precisely. So while the sample covariances will give us good estimates with oodles of data, that might not be the case with more modest sample sizes. In this case we are said to have a problem with ***weak instruments***. Solution? ***Find a better instrument!***

Example: Oregon Health Insurance Experiment[1]

[1] http://www.nber.org/oregon/ , http://www.nejm.org/doi/full/10.1056/NEJMsa1212321, and https://www.washingtonpost.com/news/wonk/wp/2013/05/02/heres-what-the-oregon-medicaid-study-really-said/